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A UNIFIED COMPARATIVE APPROACH FOR WIND INSTRUMENT PHYSICAL MODELING: FINITE ELEMENT VS DIGITAL SIGNAL PROCESSING METHODS

Dimitra Marini, Spyros Polychronopoulos, Konstantinos Bakogiannis and Georgios Kouroupetroglou

Depart. of Informatics & Telecommunications, National & Kapodistrian University of Athens, Greece
email: spyrospoly@di.uoa.gr

The simulation of musical instruments is an active field of research for both industry and academia for several decades now. We present the physical modeling methods for the simulation of the three functional parts of the wind instrument: excitation mechanism, resonator, and bell. This work highlights the different techniques that need to be used to encounter for the variations of the excitation mechanism (e.g., lip driven, air-jet, single/double reed), the resonator (e.g., cylindrical/conical, with/without toneholes), and the bell (e.g., angle). Based on the above, two models of wind instruments are simulated using Digital Signal Processing techniques and the Finite Elements Method. We demonstrate the results of both simulation techniques presenting the CPU cost/ execution time, how demanding the model is to build, and the results as a plot in the frequency domain. This work provides information that will help the community to choose the optimal method by taking into account the needs of their project.
Keywords: DSP, FEM, physical modeling.

1. Introduction

If vocal music was the first human-made music and percussion the next, wind instruments would undoubtedly be the third (and the first melody-making instrument). The idea that a dead bone or cut plant had a voice of its own was not simply considered interesting; it was magic – used to aid man with his communication with the world of spirits, to cure illness, protect crops, etc. Even though wind instruments have been played by humans for more than 30,000 years [1], the physics governing their principles of operations can still not be described in detail. Thus, it is even in our days a strong pole of attraction for many scientists.

The wind instruments are driven by an airflow coming from the musician's mouth, which when the generated pressure remains constant will, for appropriate abutment between the lips and the instrument's exciter, resulting in a harmonic oscillation of the air particles. The aftereffect of the aforementioned harmonic oscillation is a tone with a constant frequency and amplitude. While this example is convenient in order to understand how a sound with a constant pitch and amplitude is created, usually, experienced musicians purposely vary the generated sound because it is musically more interesting. The variations of control parameters, rapid transients, tremolo, vibrato, and broad-band turbulence flow noise are essential for musical sound quality; however, in this section, we will base the description of the physical phenomenon on the steady oscillation.

The realistic sonification, by digital means, of physical musical instruments, is a pole of attraction for scientists of multidisciplinary fields (i.e., physics, informatics, musicology, etc.) [2]–[5]. Theoretical acoustics has significantly contributed to the better understanding of the physical phenomenon governing the generated sound [6]–[8]. On the other hand, computer scientists have developed new cost-effective algorithms running under modern and user-friendly interfaces, which can simulate the phenomenon and sonify it in real-time [9]. These applications are not only valuable for musicians but rather to the instruments industry as well. Powerful pieces of software that, via digital simulation, sonify a virtually customized musical instrument are now commercially available, allowing accurate pre-production testing without physically building the instrument.

State-of-the-art physical modeling techniques have been put into practice by commercial companies (Native Instruments, Pianoteq, Applied Acoustic Systems, etc.) to realistically sonify various popular instruments. Various simulation techniques of woodwind instruments have been proposed [10], [11], helping towards the development of commercial pieces of software (SWAM CLARINETS, SWAM DOUBLE REEDS, SWAM FLUTES). The non-linearity introduced by the excitation mechanism and the linear effect of the oscillating air volume inside the bore, which defines the resonance and therefore the pitch of the produced sound, are the key factors that govern the physical phenomenon [12]. The physical modeling method of the Digital Waveguides (DWGs) is more commonly used, mainly because it is less computationally expensive [13].

The focus of this work is to present the physical modeling methods to simulate any wind instrument considering its three functional parts (excitation mechanism, resonator, and bell). This paper highlights the different techniques that need to be used to encounter for the variations of the: a) non-linear phenomena: the excitation mechanism (e.g., lip driven, air-jet, single/double reed) and the b) linear phenomena: the resonator (e.g., cylindrical/conical, with/without toneholes) and the bell (e.g., angle). As a case study based on the above method, two wind instruments are simulated using Digital Signal Processing (DSP). Next, using the Finite Elements Method (FEM) the same two instruments are simulated. Finally, we demonstrate the results of both simulation techniques presenting the CPU cost execution time, how demanding the model is to build, and the results as a plot in the frequency domain. This work provides information that will help the community to choose the optimal method by taking into account the needs of their project.

2. Digital Signal Processing

The built of the physical model will be described in three steps. The primary method used to implement the simulations is DWGs. DWGs is a well-established sound synthesis technique and, more specifically, a physical modeling approach. Its computational efficiency is the reason why it constitutes a significant part of modern synthesizers. This method has been developed at first by Julius O. Smith III, who has coined the term as well. Smith has filed for patents for many ideas regarding the implementation of DWG. Stanford University is the owner of the patent rights for DWG synthesis. Yamaha signed a contract with Stanford University in 1989 to jointly develop that precise technique [13]. DWGs have been under development for more than 30 years, and high-quality sound synthesis of musical instruments has been achieved. Due to the scope of this work, detailed simulations, such as involved equations, are not shown, and the reader should refer to the relevant papers in order to find more details.

2.1. Excitation Mechanism

The excitation mechanism is the only part of the wind instrument that introduces non-linear phenomena in the generation of sound. All three excitation types (shown in Figure 3) that will be discussed here can be simulated according to the relative theory underlining their acoustical behavior.

The excitation mechanism is the most challenging part of the wind instrument to thoroughly study and describe with analytical equations. The reason is not only because of the complicated non-linear physical phenomenon but also because of the significant effect on the generated sound that the lips abutment on the instrument and the saliva introduce. There have been various attempts to measure wind instruments with consistency using an artificial mouth [12], [15], but in reality, the musician's technique when playing the instrument is far too complicated to be modeled with acceptable accuracy. Usually, experienced wind instrument musicians have their own unique technique on how they use their lips. Moreover, using a technique called overblowing, the musicians can alter generated tone of the instrument up to 1,5 octaves without changing their fingering [16].

The essencial meaning of a linear theory in any domain of physics is that a) all the conclusions scale linearly with amplitude, and b) contributions from different sources are simply additive (superposition principle). As pointed out in the previous paragraph, this is not the case for the excitation mechanism. N. H. Fletcher published a paper [17] describing the main sound generation mechanisms in wind-excited musical instruments for the reed-woodwind, brass, and flute families. In this work, the theory was presented in both the frequency and time domains. The musician acts by controlling parameters of the system, which is approximately linear except for the generator, which is highly nonlinear. Fletcher argues that in order for a model to be complete, it is of utmost importance that the player is included to more accurately generate the acoustic output (see Figure 1).

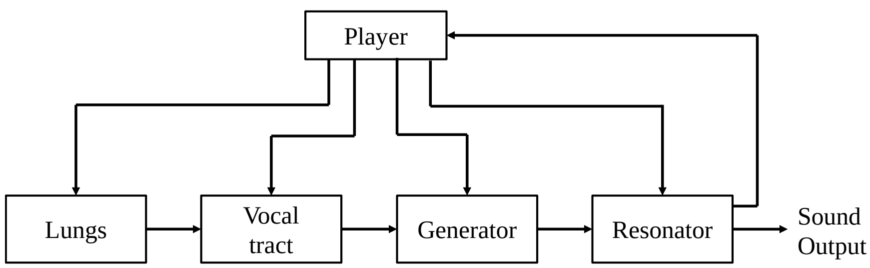


Figure 1: System diagram for a wind instrument, showing control paths and feedback paths.

In the time-domain approach, it is being assumed (which correlates to reality) that the acoustic generator in the instrument is very small compared with the acoustic wavelengths involved. Thus, the behavior can be described by a set of nonlinear differential equations that are 'local', i.e. all the variables apply to essentially the one location which, at most, subscripts to indicate whether they are input or output quantities. In the frequency-domain approach, the generator responds to the sum of the pressures or flows associated with the individual modes (nonlinear response). This response contains multiple sums and differences of the resonator mode frequencies, each with an associated phase shift, which acts as novel driving force for the resonator modes. The fact that phase shifts are involved leads to frequency shifts in the resonator modes, while cross terms between different mode frequencies ensure their interaction.

To model the generator function in a simple way, we made the following assumptions:

1. For the reed-type instruments: The reed is a linear spring, and the area of the reed aperture is proportional to the pressure difference
2. The airflow through the aperture is governed by Bernoulli's law

For the purpose of this document, in this section, we will only describe the fundamentals of the excitation mechanism. More for more details (and equations) regarding the non-linear phenomena governing the excitation mechanism in time and frequency domain, the reader should refer to [17] and more specifically for i) reed type [18], ii) air-jet type [19], and iii) lip-driven type [17].

2.2. Resonator

2.2.1. Body

In a cylindrical pipe, according to Newton's law, the relation between pressure p and volume velocity u is given by

$$\frac{\partial p}{\partial x} = -\frac{\rho}{A}\frac{\partial u}{\partial t} \tag{1}$$

where A is the pipe's cross-sectional area.

Considering a finite-length tube, the pressure waves traveling inside it experience discontinuity at each end. When a wave component encounters a discontinuous and finite load impedance Z_L at one end, a part of it will be transmitted into the discontinuous medium while the rest of it will be reflected back into the pipe. Therefore, the pressure at position x inside the pipe is given by

$$P(x,t) = \left[C^{+}e^{-jkx} + C^{-}e^{jkx}\right]e^{j\omega t} \tag{2}$$

where C^{+} and C^{-} are complex amplitudes, and the exponents $(+)$ and $(-)$ indicate the right- and left-going direction of the waves respectively (d'Alembert solution of the wave equation).

For more information regarding the calculation of the impedance [20] and the conical-type resonators [21], the reader can refer to the relative literature.

2.2.2. Toneholes

Toneholes are used to determine the exact pitch produced by the instrument. Therefore, they are among the most significant parts of the instrument, which provides the ability of a player to produce the notes that the temperament of each music genre requires. As a result, many studies describe their function. [22]–[25].

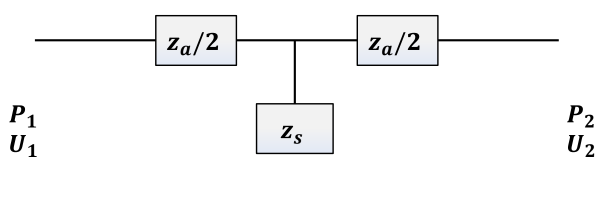


Figure 2: T transmission line representing a tonehole.

The transmission matrix of tone-hole can be approximated by a symmetric T section (see Figure 2), which depends on the shunt impedance (z_s) and the series impedance (z_a). The series impedance acts in conjunction with the longitudinal particle velocity standing wave along the axis of the main air column at the tonehole location, while the shunt impedance is related to the pressure standing wave directly under the hole. The hole impedance in series with a transitional can be derived from an analysis of symmetric pressure distribution, or a pressure anti-node, for which pressure is symmetric across the junction. [26] [23].

2.3. Bell

The bell of a wind instrument acts as a radiator of sound, which delivers the acoustic energy to the outside air and also determines the frequencies at which the resonances of the instrument will fall [27]. More specifically, the traveling waves approaching the bell from inside the bore are reflected back by the bell resulting in the creation of standing waves of precisely defined frequencies [28] (i.e., a harmonic series of resonances [27]). The bell forms a slow transition between the impedance of the tube and the low impedance of the outside air. The primary purpose of the bell is to amplify the lower frequencies. Thus, for higher frequencies, the bell is slowly changing impedance, resulting in the reflection of low-frequency waves and the transmittance of high-frequency waves.

The bore of woodwind instruments can be considered to be approximately uniform throughout most of its extent, although, at the end of their air column, sort flared sections can occur. However, it should be noted here that these sections do not significantly affect the sound propagation except for the cases when all or most of the instrument's toneholes are closed [20]. Horn and brass wind instruments, on the other hand, consist of an acoustic tube without toneholes, which terminates to a rapidly flaring bell. Therefore, in contrast to woodwind instruments where the sound primarily radiates through the tonehole lattice [20], the bell is a substantial factor of brass instruments because all sound radiation happens through it. The geometry of the bell is essential to the tuning [28] and plays an essential role in the instruments' sound quality, because the bell's physical dimensions significantly determine its acoustic behavior and reflectance properties [27].

The frequencies of the lower modes are raised because the flare of the bell makes the vibrating air column shorter at higher frequencies [29]. The influence of bore shape on the modes of typical brass instruments is described by Webster's horn equation [27]. (3) gives a one-dimensional approximation for low-frequency sound waves along a rigid tube with a variable cross-sectional area $A(x)$

$$\frac{1}{A(x)}\frac{\partial}{\partial x}\left(A(x)\frac{\partial p}{\partial x}\right)=\frac{1}{c^2}\frac{\partial^2 p}{\partial^2 t} \tag{3}$$

where x is the coordinate along the tube and c is the speed of sound [30].

The reflection and transmission characteristics of the bell are implemented by using lumped filters [27] in combination with the DWG model of the instrument. According to [31], the reflectance of the traveling waves due to the bell of a woodwind instrument is commonly modeled as a low pass filter, while the transmittance is implemented as a complementary high-pass filter. The block diagram of the sound radiation through the bell is presented in Fig. 3, using a transmittance and a reflectance filter in a DWG model implementation of a wind instrument.

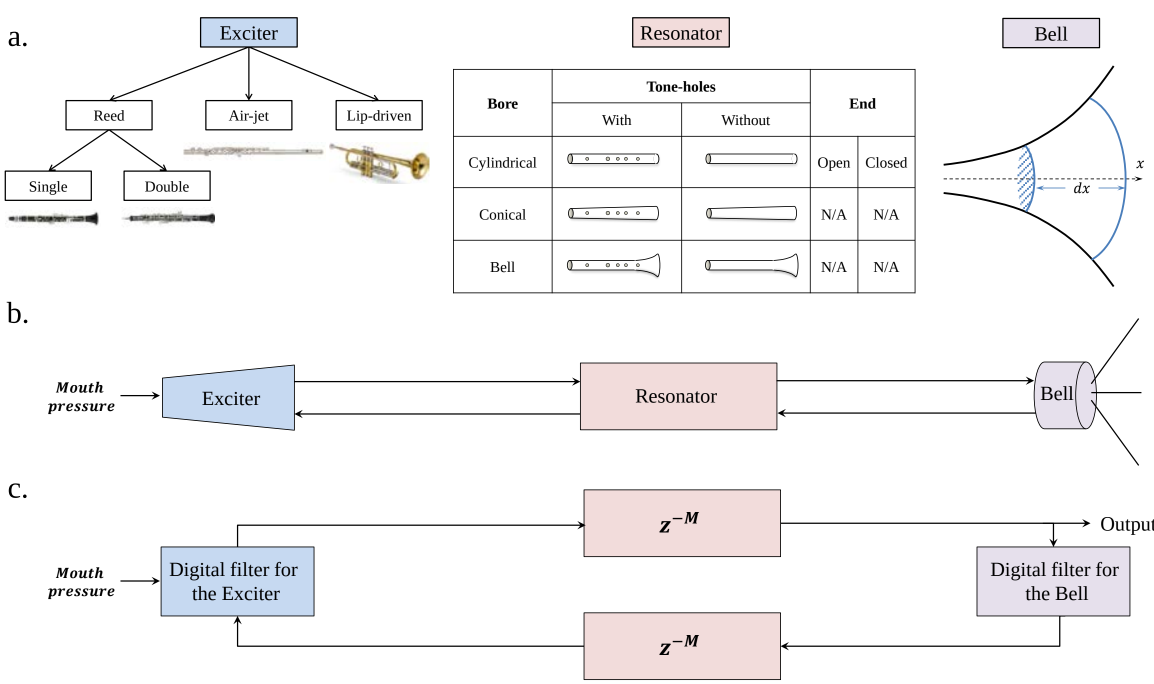


Figure 3: A typical simulation of a wind instrument. a. The 3 parts of the instrument and their variations, b. the flow diagram and c. block diagram for the digital signal processing simulation.

3. Finite Elements Method

The instrument's walls in the FEM model were simulated as hard boundary surfaces (100% reflective) in order to avoid designing small cavities (body of the instrument) where the mesh needs to be tighter and the model becomes unreasonably computational expensive to solve. This simplification will not significantly affect these models' results as pipe functions as a waveguide to drive the air oscillation; thus, the material is not a significant factor. In order to avoid the reflections on the boundaries of the solution space, we created two spheres, and the volume in between them was set to Perfectly Matched Layer (PML) element. For more details about PMLs in COMSOL (package: Pressure Acoustics) the reader can refer to COMSOL's manual [14].

Figure 4a_1 shows the mesh of a simple pipe with a conical mouthpiece and an open end, Figure 4b_1 the mesh of a recorder, and Figure 4a_2 and b_2 their 3D plot in dB at a single frequency. The mesh elements dimensions in the solving space were set to not excide the maximum frequency's the solver is calculating wavelength (here f_{max}= 2000kHz therefore, $\lambda_{max}\approx 0.17$m) over 6. Hence, the maximum element's dimension $<$ 0.17m/6$\approx$0.029m in order for the model to demonstrate accurate results. We note here that the FEM model becomes more computationally expensive as the maximum calculating frequency gets higher because the mesh needs to be tighter (smaller meshing elements).

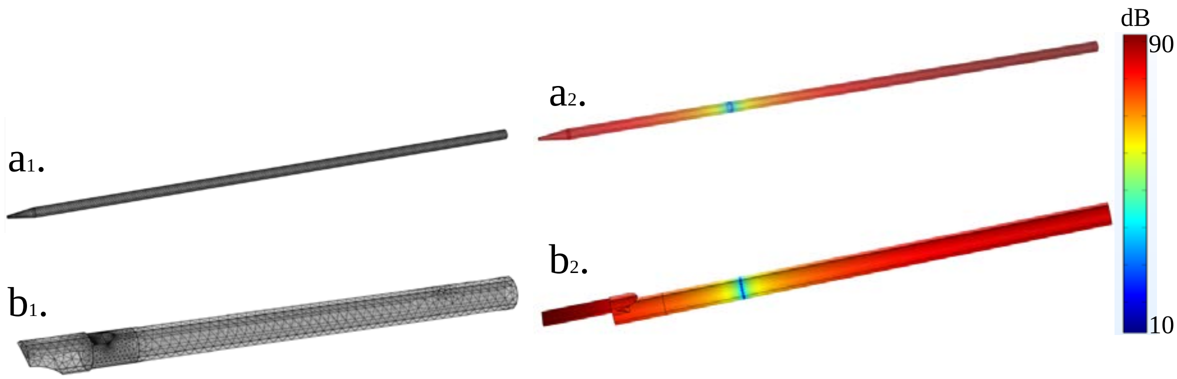


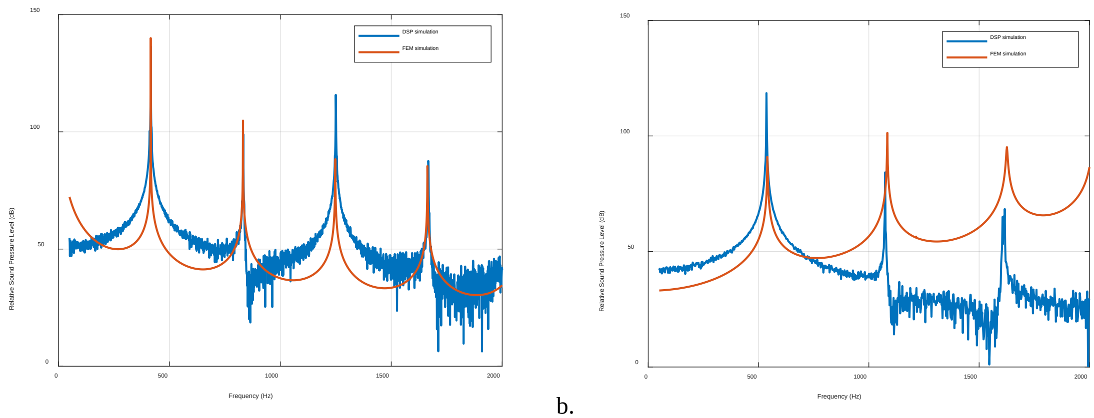
Figure 4: FEM models: a_1. the mesh of a simple pipe with a conical mouthpiece and an open end, a_2.the sound field inside the pipe (air) in dB in a single frequency, b_1. the mesh of a recorder, b_2.the sound field inside the instrument (air) in dB in a single frequency.

4. Comparison

In order to compare the signals and the execution time of the above two models (see Figure 4), we simulated both using the DSP method (see Digital Signal Processing Section) and FEM (see Finite Elements Method Section). The DSP algorithm was implemented in MATLAB 2019a, and the FEM model was built in COMSOL Multiphysics 5.5 (package: Pressure Acoustics). For both models, the bore's length is approx. 0.4m, the cross-sectional area of the opening when the reed is at rest was set to 3×10^{-6}m^2, the cross-sectional area of the bore was set to 12×10^{-6}m^2 and solving from 50Hz to 2kHz with a step of 1Hz. The frequency response of both models was calculated at the end of the bore, and it is shown in Figure 5.

The results of the two models are in good agreement regarding the fundamental resonant frequency and the bandwidth of the curves. We see that the frequency responses of the simpler model (a. simple pipe) show better agreement not only in the frequency but in relative sound pressure level as well, wherein the less simple model (b. recorder), the resonant frequencies are in good agreement, but there is a mismatch in the relative sound pressure level. As can be seen, the signal generated using the DSP method is noisy in the frequency domain, whereas the signal generated using the FEM is not.

All the models are solved using a computer running on 64-bit Windows 10 Operating System, Intel® Core™ i5-6500 CPU at 3.20GHz and 16.0 GB of RAM. As was expected, the execution times for both models (a. simple pipe and b. recorder) were greater using FEM than using the DSP method (see Table 1).



a. b.

Figure 5: Comparison of signals in the frequency domain of two models, a. simple pipe and b. recorder using two methods: DSP (MATLAB 2019a) digital waveguide method plot in blue and FEM (COMSOL 5.5) plot in blue.

Table 1: Execution times of the physical models simulated in DSP and in FEM

Simulation Method	a. Simple Pipe	b. Recorder
DSP	10sec	50sec
FEM	25mins	35mins

5. Discussion

The simulation of two models of wind musical instruments (simple pipe and recorder) was implemented using DSP and FEM techniques. All the simulation steps are easy to follow, and relative literature is also provided in order for this work to trigger further research to compare the DSP and FEM results with recordings of physical instruments. As the geometry gets complicated, we expect the FEM to work more efficiently, but the computational time is rather crucial for most applications, and that is the weakest point of this method.

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