An efficient filter-based model for calculating absorption coefficient and directivity of resonator panel absorbers

By Spyros Polychronopoulos

Historical background

A resonator is a cavity with one or more apertures which have the property to resonate in certain frequencies. Our current knowledge regarding its functions is not complete. Thus, a great number of scientists worldwide focus their research on this field. Aristotle makes the first clear reference to a resonator in the 4th century B. [1]. It is probably the first acoustic element used in architectural acoustics. A few centuries later, Vitruvius (1st century AD) refers to resonators that were placed under the seats in ancient Greek and Roman theatres to improve their acoustic quality [2]. Later, in medieval ages resonators were embedded inside the walls of worship spaces for the same purpose [3-6].

Nowadays, the common resonator is known as the Helmholtz resonator, taking its name from the German physicist Hermann von Helmholtz (1821-1894) [7]. The theory of Helmholtz resonators was firstly established by J W S Rayleigh (1842-1919) [8], and half a century later KU Ingard described analytically their function and use as sound absorbers and scattering devices [9].

Introduction/basic theory

The Helmholtz resonator is a lumped element that has the property, due to its shape, to attenuate acoustic energy at its resonant frequency in the far field. The resonant frequency depends upon the geometry of the resonator. Accurate, but rather complicated, formulas can be used to calculate the resonant frequency for a variety of different shaped resonators and be found in bibliography [10-14]. The first approximation for calculating the resonance frequency, which was introduced by Ingard, is given by the following formula [9]:

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S}{VL}} \qquad (1)$$

Where c (m/s) is the speed of sound, S (m^2) the cross section area of the neck, L (m) is the effective length of the neck (including the end corrections), $L=L'+1.7a$, L' (m) is the real length and V (m^3) the volume of the cavity. The geometric characteristics can be seen in Figure 1.

The operation of a Helmholtz resonator can be identified via its mechanical and electrical equivalent systems and it is shown in the following figure.

Helmholtz resonator as a filter

The acoustic performance of an ideal Helmholtz resonator can be simulated as a second order system, whose discrete time domain impulse response can be modelled by an IIR filter [15], i.e. a filter having both feed-forward and feedback terms. Such a digital filter, for an ideal Delta function input *δ(n)*, will yield as output its impulse response, and for any input *x(n)*, the output *y(n)* will be represented as shown in the following block diagram, where n is the sample. Figure 3 shows a block diagram of this IIR filter while Figure 4 shows the impulse response of the filter in time and frequency domain.

Perforated panel as a filter

Modern perforated panels, as shown in Figure 5a, are commonly

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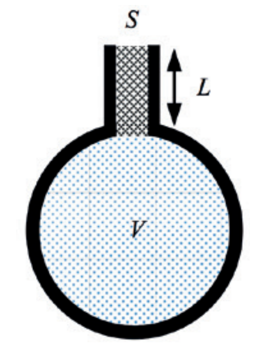

Figure 1. The geometry of a simple Helmholtz resonator

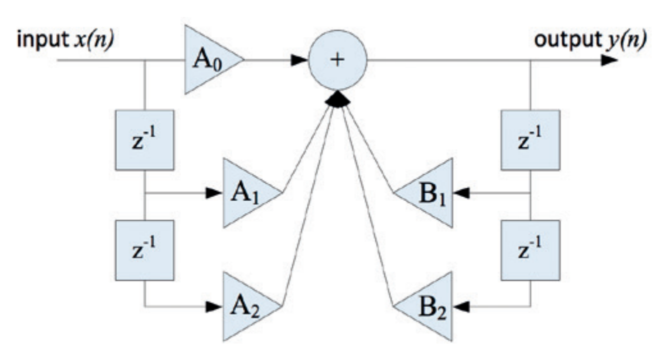

Figure 3. Block diagram of a second order IIR filter

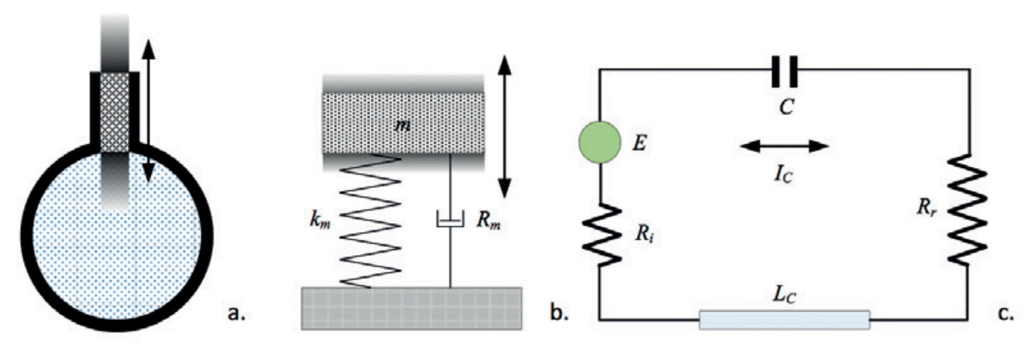

Figure 2. a. Helmholtz resonator in operation, b. Mechanical analogue and c. Electrical analogue

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used to improve the acoustics of internal spaces. This element is an array of Helmholtz resonators installed on a surface. Assuming that each resonator can be simulated as a multidirectional point source, the signals that will reach the receivers' point, when the system is given an input, are shown in Figure 5b.

For the sound reflected from the panel, taking into account the finite distance differences from the source to each resonator in the panel, the specific time delay and the sound pressure attenuation are calculated for each resonator element, so that the input sound that reaches each resonator is attenuated by

$$\frac{p_A}{r_{ABi}}$$

and delayed by n_{ABi} samples, where i is the index number of each Helmholtz resonator, A and B the source and receivers' point respectively and p_A the source sound pressure. In the next step the incoming sound pressure is filtered by the Helmholtz resonator filter and the output sound pressure from each resonator is represented by an omnidirectional wave, emitted to the receiver (green dashed line, Figure 5b).

Due to the summation of N elements for the Helmholtz resonator path, the overall level of the response increases disproportionally with respect to the level of the reflection that would be received due to a normal panel. The calibration factor (c_f), adjusts the peak level of the combined panel and Helmholtz resonators to the level of a non-perforated panel. The complete block diagram of the processing steps followed is shown in Figure 6.

Typical time and frequency domain response results, for a 4x4 Helmholtz resonators' array attached to a panel, are shown in Figure 7. For this example, the resonator's filter parameters were set so the resonance frequency was to be 115Hz, whereas the panel surface with typical absorption coefficient was assumed in order to illustrate more clearly the panel's material effect.

Filter based resonator panel model in diffuse field/calculating the absorption coefficient

The previous section was concerned with the filter-based

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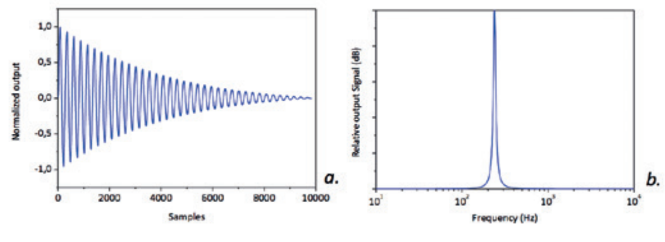

Figure 4. Impulse response of the second order IIR filter:
a. in time domain (samples) and b. in frequency domain

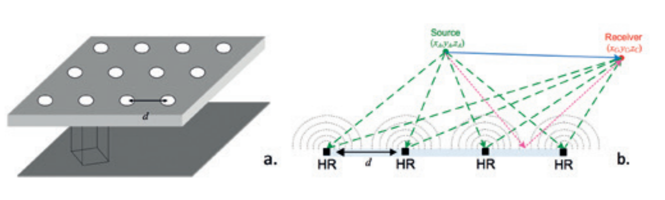

Figure 5 a. Perforated panel and 5b. Signal paths from source to receiver for a perforated panel in free field

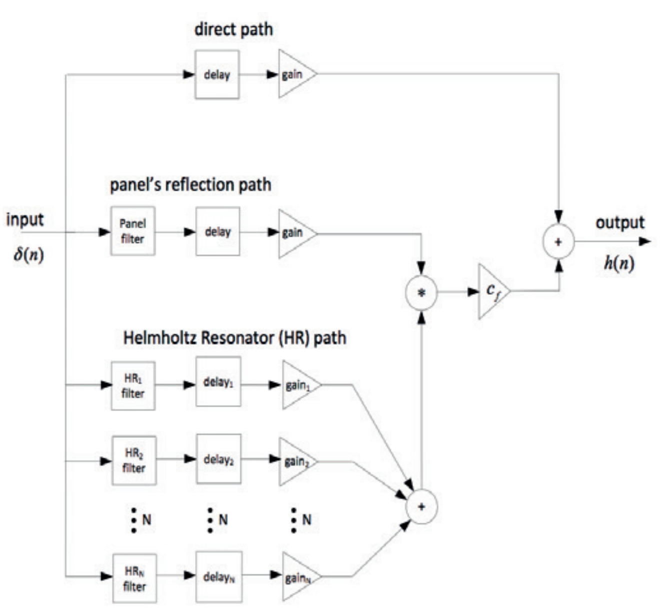

Figure 6. Block diagram for the proposed filter-based model of an absorber panel containing an array of N Helmholtz resonators. The ideal free field model is shown here, whereas for the diffuse field case, the input will be the ideal diffuse field impulse responses and the direct path will be omitted

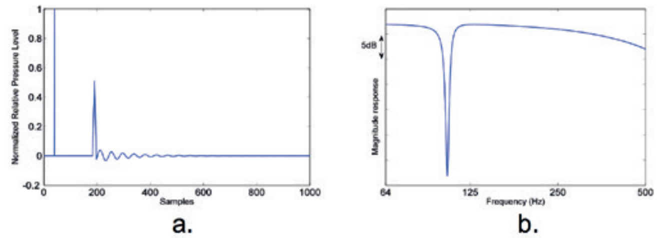

Figure 7. Impulse response at receiver position for a combined 4x4 Helmholtz resonators array and panel due to a delta impulse at source position. a. Impulse response and b. Magnitude frequency response

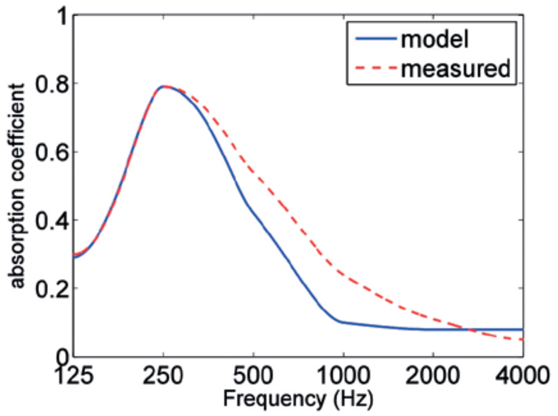

Figure 8. Typical comparative results for absorption coefficient evaluated for 1/3 octave bands of a commercially available resonator absorption panel and the simulated data of the same panel obtained by the proposed method

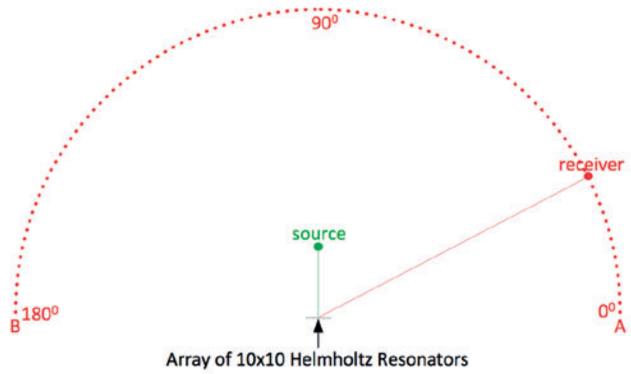

Figure 9. Helmholtz resonators 10x10 array with displaced receiver along an arc

evaluation of the resonator absorber panel's response in the free field. However, the absorbing properties of such panels need to be measured via the reduction of sound energy achieved under ideal diffuse field conditions, as is dictated by ISO 354 [16]. Following the proposed model such a condition will be realized, assuming that the sound input of the simulated panel system is not an ideal Delta function, but instead an ideal diffuse field response. By definition, such an impulse response will be characterized by randomly arriving reflections, decaying exponentially with time. Such diffuse components of room impulse responses may be modelled by exponentially decaying white noise [17].

Hence, by driving the panel filter model via such diffuse field response corresponding to an ideal reverberation chamber, the filtered output will emulate the measured response at a specified receiver position, when the absorber panel is installed. The modelling procedure will also follow the block diagram shown in Figure 6, where in case the input is the ideal diffuse room the output will be the response.

In order to evaluate the absorption properties of panels modelled according to the proposed approach, an ideal diffuse reverberant chamber is simulated. Its impulse response (arriving at the resonator-panel system) is evaluated for arbitrarily chosen value of the reverberation time. Then, any resonator absorption panel of specified parameters (perforation spacing, resonant frequency, panel material and size) is assumed to be placed inside such space, resulting in to a modified response at a receiver position and now having a modified reverberation time due to increased absorption. The simulated tests here attempt to predict the absorption coefficient of the panel system under such ideal diffuse field conditions, as is dictated by the standardized procedure.

The comparative 1/3 octave results, for the measured and estimated absorption coefficient that are shown in Figure 8, indicate a close approximation between measured and simulated data, especially around the resonant frequency region. In this simulation, some discrepancy is observed at higher frequencies due to inexact representation of the specific plain surface absorption of the specific panel. Such discrepancies can be reduced when the exact properties of the panel material are known prior to modelling. For more analytical description and formulas please see [18].

The signals' contribution as a factor of receiver's angle

In order to study the effect of receiver's angle to a Helmholtz resonators array and be able to plot a polar diagram, the arrangement as shown in Figure 9 was modelled. The following figure depicts an array of 10x10 resonators, a displaced receiver along an arc by 1° step and a point source that stays constant. The source emits an ideal impulse (Delta function δ).

In Figure the 10 red line depicts the case of one resonator. It is observed that the directivity is independent of the angular displacement of the receiver as provided theoretically [19]. The light green line stands for resonators distance of d=0.02m, the blue line for resonators distance of d=0.04m and the darker green line for resonators distance of d=0.08m.

By comparing the figures, it is observed that at the resonant frequency the acoustic field is significantly reduced in all cases. It is also shown how the degree of perforation of perforated panels affect the directivity.

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Conclusion

A novel and computational efficient method for evaluating the response of perforated absorption panel with an arbitrary number of Helmholtz resonator elements has been proposed, based on a simplified, parametric filter-based model. The combined response of these resonators and of a panel surface having user-defined reflection properties can be also predicted, leading to the efficient evaluation of the system's impulse and frequency response functions. An extension of the method allows the evaluation of the response of such a system under ideally diffuse acoustic excitation and an efficient estimation of the resulting time, energy and frequency response. From those functions, the reduction of the initial diffuse field reverberation time due to the panel absorption can be evaluated, leading to the estimation of the frequency-dependent absorption coefficient of the simulated panel specimen according to the ISO standard. This was confirmed by a comparative test between simulated results obtained by the proposed method and published measurement data derived from a standardized test of a commercially available perforated panel.

Analytic solutions for systems and applications of complexity such as of the ones covered by the proposed approach, are beyond the capabilities of current computer systems. The proposed method, implemented in Matlab [20], introduces a flexible and practical alternative having far shorter and manageable computation time requirements than any Finite Elements Method (FEM) - based method. The following table shows the typical CPU computation time as function of N resonator elements for the numerical evaluation of resonator panel response using a FEM-based model (using Comsol Multiphysics [21]) and the proposed filter – based model. Note that from the publication date of the paper [18], a more efficient algorithm reduced the computation time even more, as shown in the table below.

	Number of Resonators			
	1	16	64	2500
FEM-based model	4 hours	17 hours	2 days	Not feasible
Filter-based model	5 sec	20 sec	2 min	18 min

Table 1. Typical CPU computation time as function of N resonator elements for the numerical evaluation of resonator panel response using a FEM-based model and the proposed filter – based model.

As discussed previously the proposed method is useful for estimating the distance and angle depended response of any arbitrary-sized perforated panel surface, not only having a given perforation ratio, but also potentially having any non-symmetric distribution of the resonator element array. Besides, the resonator elements may be unequally-sized, potentially leading to novel and optimized solutions for sound absorption.

***Spyros Polychronopoulos** was born at Athens in 1980. Even from his youth, he was interested in sound as a physical phenomenon as well as in sound's artistic perspective (music). After his graduation from the Physics Department, he completed his PhD in Polytechnic department of University Of Patras on acoustics and he has published a number of papers. As for the artistic aspect of sound, he released 14 albums and conducted many concerts. He has also organised workshops and gave lectures regarding the new technologies in composition and aesthetics of music. He works at KP Acoustics, London.*

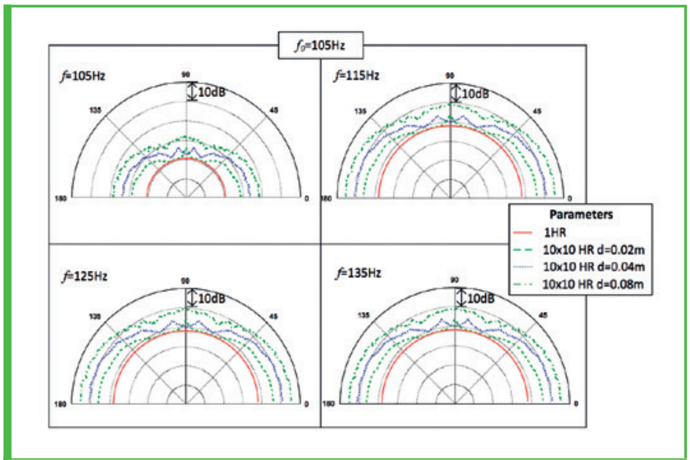

Figure 10. Polar diagram for various distances between the Hemholtz resonators and various frequencies (f=105Hz, f=115Hz, f=125Hz and f=135Hz) for resonance frequency f_0=105Hz

References

1. Aristotle, "*Problems*", original text 4th century B.C., Edited and translated by Robert A. Mayhew, Loeb Classical Library, 2011.
2. Marcus Vitruvius Pollio, "*The Ten Books on Architecture De Architectura*", original text 1st century A.C., translated by M. H. Morgan, Harvard University Press, 1914.
3. Mijic M., Sumarac- Pavlovic D., "*Analysis of contribution of Acoustic resonators found in Serbian orthodox churches*", Building Acoustics vol. 11, 2004.
4. Zakinthinos T., Skarlatos D., "*The effect of ceramic pots on the acoustics of old Greek orthodox churches*", Applied Acoustics vol. 68, Issue 11-12, 2007.
5. Spyros Polychronopoulos, Dimitris Kougias, Polykarpos Polykarpou and Dimitris Skarlatos, "*The Use of Resonators in Ancient Greek Theatres*", Acta Acustica united with Acustica Vol. 99, 2013.
6. Panagiotis Karampatzakis, Vasilis Zafranas and Spyros Polychronopoulos, "*A study on Aristaxenus acousic urns (The Vitruvian secret)*", Europian Acoustics in association with Hellenic Institute Of Acoustics, 2011.
7. Hermann von Helmholtz, "*On the Sensations of Tone as a Physiological Basis for the Theory of Music*", translated by Alexander John Ellis, Longmans Green and Co., 1875.
8. J. W. S. Rayleigh and Robert B. Lindsay, "*The theory of sound*", original text 1877, Dover Publication, 1945.
9. K.U. Ingard, "*On the theory and design of acoustic resonators*", Journal of the Acoustical Society of America, 1953.
10. C. L. S. Gilford, "*Helmholtz resonators in the acoustic treatment of broadcasting studios*", British Jurnal of Applied Physics, 1951.
11. M.Alster, "*Improved calculation of resonant frequencies of Helmholtz resonators*", Journal of Sound and Vibration, vol. 24 no 1, 1972.
12. Panton R. και Miller J., "*Resonant frequencies of cylindrical Helmholtz resonators*", Journal Acoustical Society of America, Volume 57, 1975.
13. Li D., "*Vibroacoustic behaviour and noise control studies of advanced composite structures*", PhD Thesis at University of Pittsburch, 2003.
14. S.K. Tang, "*On Helmholtz resonators with tapered necks*", Journal of Sound and Vibration 279, 2005.
15. Ken Steiglitz, "*A Digital Signal Processing Primer: With Applications to Digital Audio and Computer Music*", Prentice Hall, 1996.
16. ISO 354: "*Acoustic – Measurement of sound absorption in a reverberation room*".
17. Jean-Marc Jot, Laurent Cerveau and Olivier Warusfel: "*Analysis and Synthesis of Room Reverberation Based on a Statistical Time-Frequency Model*", 103rd Audio Engineering Society Conv., New York, 1997.
18. Spyros Polychronopoulos, Dimitris Skarlatos and John Mourjopoulos, "*Efficient filter based model for Helmholtz resonator panel absorbers*", Journal of Audio Engineering Society, Volume 62,p. 14-24, 2014.
19. E. Kinsler, Austin R. Frey, Alan B. Coppens και James V. Sanders, "*Fundamentals of Acoustics, Fourth Edition*", John Wiley & Sons, Inc., 2000.
20. MATLAB and Statistics Toolbox Release 2012b, The MathWorks, Inc., Natick, Massachusetts, United States.
21. Comsol Multiphysics, www.comsol.com, Acoustic module v3.4