Efficient Filter-Based Model for Resonator Panel Absorbers

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An efficient method for simulating the far field acoustic radiation from resonator panel absorbers is presented following a filter-based modeling approach. This method allows the evaluation of the time and frequency domain response of arbitrary-sized perforated panels in any specific receiver position under free-field and diffuse-field acoustic environments. Such panel absorbers employ the Helmholtz resonator principle, whose filter-based representation is well-defined. Hence, combining such basic resonator elements, the physical parameters of the panel can be user-defined in a parametric fashion, and it is shown that from the derived impulse response, the panel's absorption coefficient can be evaluated with sufficient accuracy, following the standardized procedure. Unlike existing analytical approaches, the proposed approach offers significant computational efficiency and allows flexible and fast practical evaluation of the effect of such panel absorbers.

0 INTRODUCTION

During the last decades, an increasing number of acoustic signal propagation cases have been analyzed and described by treating the transmission medium and boundary elements as linear filters and utilizing the time-to-frequency Fourier transform mathematical framework. Such approach describes the response (impulse response or Green's function) of the system at time t and the spatial location of the receiver C (x_C, y_C, z_C) due to a unit-amplitude impulse applied at instance $t_0(t=0)$ at the spatial position of the source, e.g., at A (x_A, y_A, z_A), the impulse response being typically expressed as $h(t,r_{AC})$, or for simplicity as $h(t)$. For most such acoustic applications a causal time-invariant, space-invariant description of the system is utilized, deriving the output $y(t)$ at the receiver position due to the application of a signal $x(t)$ at the source position, via its convolution with such a unique defined impulse response $h(t)$ [1].

The advantages of such an approach are mainly related to the efficiency and flexibility in simulating and studying the acoustic system for varying physical parameters. In room acoustics, such a well established model is the image method of Allen and Berkley [2]. It is here proposed to adapt such simplified but computational efficient models to the analysis of common acoustic materials such as perforated absorber panels so that these can be integrated to widely employed room acoustic simulation methods. The proposed method is not aiming at an exact analytic description of the sound-field generated by the acoustic radiation of such materials; but instead it introduces a flexible computational framework that can derive their response to any acoustic signal at a specified receiver position.

With respect to such materials employed for modifying room acoustics, when energy absorption needs to be applied in a specific narrow frequency band, usually associated with the low frequency region, Helmholtz resonators provide a well established solution [3]. Helmholtz resonators are not distributed acoustic elements. Thus, an absorption coefficient cannot be directly calculated, since it always refers to a uniform surface. Instead, their absorbing power is characterized by their "absorption cross section" or their equivalent absorption area, which is defined as the ratio of sound energy being absorbed per second by such elements and the intensity, which the incident sound wave would have at the place of the absorbent object if it was not present. Typically, the main function of resonators is their sound absorption with a large absorption cross section at their resonant frequency. Helmholtz resonators also have a remarkable scattering cross section and under optimum conditions their absorption cross section has a maximum equal to $\lambda^2/4\pi$ [4,5].

A perforated panel utilizing such resonating elements is a common material for sound absorption, with a wide use in room acoustics treatment applications [3]. Although the resonant frequency of a single resonator can be calculated by either simple or more exact analytic formulae, the basic problem of panel absorbers is the calculation of their absorption coefficient, since only approximate values can be derived with the existing formulae. On the other hand, more accurate estimation can be obtained with the use of physical modeling methods, as for example Finite Elements Method (FEM) [6,7, 8,9, 10] and Finite Time Difference Method (FTD). Although such work often realized via commercially available simulation packages [11,12] has significant advantages in terms of analytic exactness, currently it can be limited due to the excessive computational complexity required. For example, solving a 3-D model for a single resonator with an FEM in frequency domain, a current computer might need over a day of computational workload. Furthermore, such FEM models mainly generate results in the frequency domain, so all of the acoustic parameters or linear filter time domain response components, cannot be directly evaluated.

As is well known, the presence of a perforated panel absorber affects strongly the room acoustic indices, and their main effect is manifested on the reverberation time. This feature is traditionally used for the random incidence absorption coefficient estimation under ideal diffuse reverberant conditions following the ISO 354 standard [13]. To the best knowledge of the authors there is no current efficient analytic or computational model that can predict the absorption behavior of such panels as is required by the ISO 354 standard. Instead, the measurement of their acoustic properties needs to be performed following elaborate testing procedures in large reverberant chambers.

Given the limitation of the above analytic methods for modeling such absorbers, a new method is proposed here, based on a simplified but efficient linear filter approach that describes the point-to-point time (impulse) response and corresponding transfer function of acoustic panels consisting of resonant absorbers. The proposed method follows many simplifications with respect to the physical mechanisms of acoustic radiation from such resonator panels, as well as the properties of the acoustic excitation and as such is not aiming at an exact analytic description of the sound field generated under such applications. Instead, the proposed method allows flexible definition of resonator parameters that can be arbitrarily arranged on perforated panels. As will be shown, this approach allows efficient investigations to be undertaken for the evaluation of the absorption coefficient of such panels following the current standardized procedure. Other studies and simulations may also be undertaken for different applications since these can be directly integrated with widely-used current room acoustics simulation techniques. Specifically, it will be shown that the proposed computationally efficient method can derive the time and frequency domain response of any $N_x \times N_y$ elements absorber panel at any receiver position under ideal anechoic conditions and also under ideal diffuse field conditions such as required by the ISO 354 standard. Under such conditions, the method allows sufficiently accurate estimation of the frequency dependent absorption coefficients of such panels to be evaluated, as was found after comparison to measured results for commercially available panels.

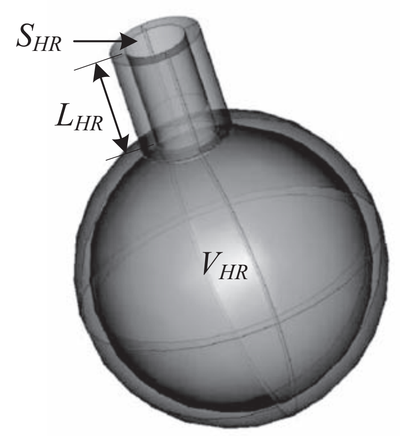


Fig. 1. Single Helmholtz resonator geometry

The paper is organized as follows. Section 2 discusses the proposed modeling approach and, specifically, Section 2.1 describes the basic principles of Helmholtz resonator acoustic properties; Section 2.2 describes the filter-based modeling of a single Helmholtz resonator attached to a panel; Section 2.3 analyzes the proposed Helmholtz resonator model as a parametric filter; Section 2.4 describes the proposed model for a resonator panel consisting of *N* elements in free field; Section 2.5 describes the proposed model of the resonator panel in diffuse field; Section 2.6 describes the way the absorption coefficient of the resonator panel is calculated utilizing the model that was described in the previous Sections. Section 3 presents typical results based on simulations using the proposed model and specifically Section 3.1 presents a comparison between estimates for the absorption coefficient of a specific panel derived by the proposed method and data derived by the standardized measurement; Section 3.2 evaluates the computational complexity of the proposed model as opposed to a similar, based on the FEM method. Finally, Section 4 presents a discussion and conclusions derived by this work.

1 MODELING APPROACH

1.1 Helmholtz Resonator Principles

The Helmholtz resonator (Fig. 1) is a lumped element that has the property, due to its shape, to attenuate acoustic energy at its resonance frequency in the far field. For a simple resonator, its resonant frequency can be expressed with good approximation by the following equation [4,14]:

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S_{HR}}{L'_{HR}V_{HR}}} \tag{1}$$

Where c is the speed of sound (in m/s), S_{HR} orifice surface (m^2), L'_{HR} the length of the neck including the end correction (in m) and V_{HR} the volume of the body (in m^3). The end correction is a correction factor, added to the neck's length $L'_{HR} = L_{HR} + 1.7r_{HR}$, where r_{HR} is the radius of the Helmholtz resonator's embouchure (in m). More

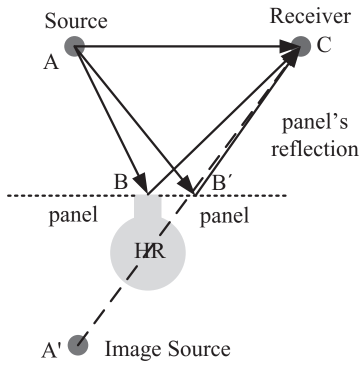


Fig. 2. Free field representation of Helmholtz resonator (HR) model attached to a panel with signal paths shown.

accurate but more complicated equations are available as well [3,15].

1.2 Filter-Based Helmholtz Resonator Sound Field Model

Let us assume an ideal omnidirectional point sound source at point A (x_A, y_A, z_A) that emits a sound wave of pressure $p_{source}(r, t)$, where r is the distance and t the time. This sound wave reaches both the receiver (located at point C, following the direct path from point A to point C) and a single resonator (from point A to point B) it is attenuated and delayed due to the distances AC and AB respectively as the sound wave travels in the air medium, as shown in Fig. 2. The sound wave reaches the embouchure of the resonator and then the resonator re-emits the signal back to the sound field attenuated at the resonator's resonance frequency. Note that in its near field, generally for distances in the region of $r < \frac{c}{\omega_0}$, the sound pressure at the resonance frequency is amplified [16,17]. Given that here we focus on the behavior of the resonator in the far field, its near field behavior will not be considered. So, for the cases studied here, the receiver is assumed to be at distances in the region of C $r > \frac{c}{\omega_0}$ (i.e., at the far field), so that the resonator will act as a sound absorber.

Although the present analysis can be extended to a higher frequency range, here we shall consider typical cases when the resonance frequency of the Helmholtz resonator is lower than 500 Hz ($f_0 < 500$ Hz), as is the case for those used in most perforated panel absorbers. A typical radius of the embouchure for this resonance frequency (see Eq. (1)) is $r_{HR} < 0.005$ m, in order that the length of the neck and the volume of the resonator to be within the practical panel size. Thus, in this case the resonator is assumed to behave as an omnidirectional point source, since $kr_{HR} \ll 1$ [18], where $k = \frac{2\pi f_0}{c}$.

The Helmholtz resonator can be now modeled as a time domain filter assuming that $p_{HRin}(t)$ is the sound pressure that reaches the embouchure and $p_{HRout}(t)$ is the output pressure emitted by the resonator. Therefore, the Helmholtz resonator is approximated as an omnidirectional point sound source and the re-emitted signal from the resonator reaches the receiver (from point B to point C) being also attenuated and delayed in relation to the distance BC. The total acoustic pressure at the receiver point in this free field case will consist of the combined contributions both from the direct sound pressure from the source and the sound pressure emitted by the Helmholtz resonator (see Fig. 2).

Now it is further assumed that the resonator is located on a panel as is the typical case for perforated panel absorbers. The contribution from the panel's surface will be here approximated as an additional linear filter component also characterized by an impulse response $h_P(t)$. The plain panel response (that is, its response without any resonator perforation) can correspond to measured results for the common acoustic materials employed, but here $h_P(t)$ will be simplified and will be approximated by an angle-independent response whose spectrum corresponds to typical reflectivity pattern of the simulated panel material. Assuming that $p_{HRPout}(t)$ correspond to the pressure response from the combined resonator and panel system, and taking into account the relative time delays and pressure attenuation of the sound traveling in the air medium, the combined sound pressure at the receiver point C will be:

$$p_C(r,t) = \frac{p_{source}(r,t)}{r_{AC}} e^{j(\omega t_{AC} - kr_{AC})} + \frac{p_{HRPout}(r,t)}{r_{BC}} e^{j(\omega t_{BC} - kr_{BC})} \tag{2}$$

In the following analysis it is assumed that the relative delay imposed by the path reflection AB'C evaluated by the image A' of the source (see Fig. 2) will be minimal and thus can be ignored, so that only the linear filtering effect of $h_P(t)$is considered.

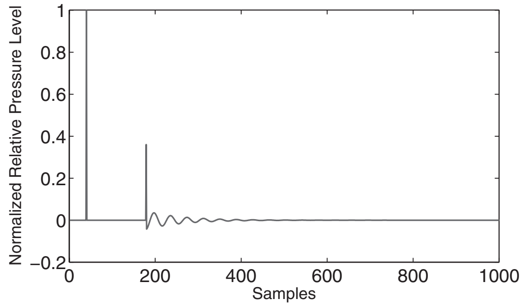
To extend the linear filter approach, it is now convenient to consider a case where the pressure at the source (at point A) is an ideal delta function, i.e., $p_{source}(r, t) \cong \delta(r, t)$ and after simplification for position-specific case and by adopting a discrete time notation, $p_{source}(n) \equiv \delta(n)$. Note that n is the discrete time sample index, for convenience ignoring the sampling period T_s. Then, the pressure of the combined system at the receiver position C, will correspond to the discrete time impulse response $h_{HRC}(n)$, as will be shown in more detail in Fig. 6. When the combined resonator and panel system is considered, leading to a combined response $h_{HRPC}(n)$, then, the discrete time impulse response at the receiver position C will also contain the direct path component $h_{AC}(n)$, i.e.:

$$h_{HRC}(n) = h_{AC}(n) + h_{HRPC}(n) \tag{3}$$

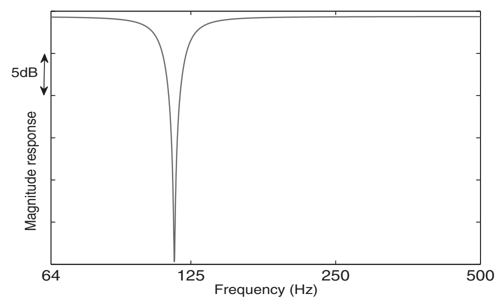
Typical results following such a model, for the impulse response of the system (evaluated as will be discussed in detail in Section 2.3) and the corresponding magnitude transfer function$|H(\omega_k)|$, are shown in Fig. 3. Note that the discrete frequency response at any specific point (e.g., at C) is estimated via the DFT of $h_{HRC}(n)$, i.e.:

$$H_{HRC}(\omega_k) = \sum_{n=0}^{L-1} h_{HRC}(n) e^{-j\omega_k n}$$

where ω_k (rad/s) indicate the discrete frequency index and L (samples) the length of the response signal. Typically, the responses shown in dB will refer to the log



(a)



(b)

Fig. 3. Typical response at receiver position C (0, 1, 2) for a single Helmholtz resonator at B (0, 0, 0) due to a delta impulse at source position A (0, 0, 1) and for $f_0 = 120$ Hz. Note that in this simulation the reflection from the panel was ignored. (a) Impulse response, (b) Magnitude frequency response.

magnitude 20log $|H_{HRC}(\omega_k)|$of the above function and are implemented via FFT. Similar transformations will apply to all other relevant signals discussed in subsequent sections.

2.3. Helmholtz Resonator Parametric Filter Model

An ideal Helmholtz resonator represents a second order system whose discrete time domain impulse response $h_{HR}(n)$ can be modeled by an IIR filter [19], i.e., a filter having both feed forward and feedback terms. Such a digital filter, for an ideal Delta function input $\delta(n)$, will yield as output its impulse response, $h_{HR}(n)$ and for any input $x(n)$, the output $y(n)$ will be represented by the difference equation:

$$y(n) = A_0 x(n) + A_1 x(n-1) + A_2 x(n-2) - B_1 y(n-1) - B_2 y(n-2) \quad (4)$$

Hence, in this equation, if the input $x(n)$ describes the incident sound pressure $p_{HRin}(n)$ (see Section 2.2) and the coefficients A_0, A_1, A_2, B_1, and B_2 describe the parameters of the resonator evaluated as functions of its geometry, then $y(n)$ will be the output sound pressure $p_{HRout}(n)$. Thus, when the input to the resonator is assumed to be an ideal Delta function $\delta(n)$, then the impulse response $h_{HR}(n)$ of the Helmholtz resonator can be written as:

$$h_{HR}(n) = A_0\delta(n) + A_1\delta(n-1) + A_2\delta(n-2) - B_1\delta(n-1) - B_2\delta(n-2) \quad (5)$$

For any such resonator filter, the coefficients A_0, A_1, A_2, B_1, and B_2, can be directly and efficiently related [20,21] to the filter parameters (center frequency, passband gain, and bandwidth), through the expressions:

$$A_0 = \frac{1 + a_p + k_{gain} - k_{gain}a_p}{2} \quad (6)$$

$$A_1 = b_p(1 + a_p) \quad (7)$$

$$A_2 = \frac{1 - k_{gain} + a_p + k_{gain}a_p}{2} \quad (8)$$

$$B_1 = A_1 \quad (9)$$

$$B_2 = a_p \quad (10)$$

$$a_p = \frac{1 - \tan\left(\frac{\pi B}{F_s}\right)}{1 + \tan\left(\frac{\pi B}{F_s}\right)} \quad (11)$$

$$b_p = -\cos\left(\frac{f_0}{F_s}\right) \quad (12)$$

F_s (samples/sec) is the sample rate. Note that at for all of the following simulations $F_s = 44100$. B (Hz) is the bandwidth can be calculated from the following equation [22]:

$$B = \frac{f_0}{Q}(\text{bandwidth in Hz}) \quad (13)$$

Q being the Helmholtz resonator quality factor, calculated [22] from:

$$Q = \frac{2\pi f_0 M_A}{R_A} \quad (14)$$

Factors M_A and R_A can be calculated [22] as:

$$M_A = \frac{\rho_0 L'_{HR}}{\pi r_{HR}^2} \text{ in } {}^{kg}\!/_{m^4},\ \rho_0 \text{ is the medium's density}$$
$$\rho_0 \simeq 1,2 {}^{kg}\!/_{m^3} \text{ for air medium} \quad (15)$$

$$R_A = \frac{\sqrt{4\pi f_0 \rho_0 \mu}}{\pi r_{HR}^2} \frac{L_{HR}}{r_{HR}} \frac{16}{3\pi} \text{ in } g/m^2 \text{ sec}, \quad (16)$$

μ is the medium's viscosity $\mu \simeq 1,8 \cdot 10^{-5}$ for the air medium.

Assuming the filter acts as attenuator (at the far field), hence having a negative gain, then the attenuation at the resonance frequency can be calculated in dB as:

$$k_{gain} = Q \quad (17)$$

2.4. Filter-Based Resonator Panel Model in Free Field

Following the time domain model of the single resonator and panel described in the previous section, an array of Helmholtz resonators (HR) arranged on a panel surface can be emulated leading to an efficient representation for the response of such panel absorber. Although, in principle any combination of resonators with different physical properties and hence filter parameters can be arbitrarily arranged on a 2-D panel and may be evaluated following this approach, here for simplicity, Helmholtz resonators of identical properties will be placed symmetrically at equal distances (d_{HR}) one from another, as shown in Fig. 4 creating an array of

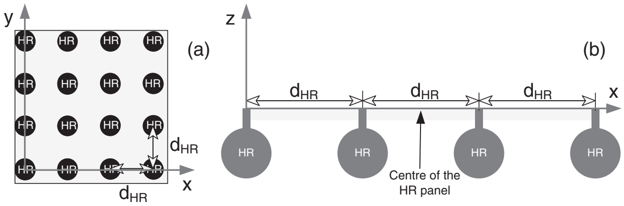


Fig. 4. Representation of a 4×4 resonator element array panel: (a) plan view, (b) side section.

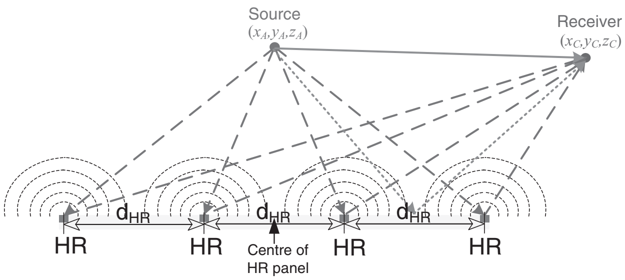


Fig. 5. Free field representation for the response of a 4 × 4 Helmholtz resonator array attached to a panel with signal paths shown.

$N_x \times N_y$ elements. Here N_x, N_y is the number of Helmholtz resonators in each dimension (*x, y*). For simplicity, a square panel here is analyzed, so for $N_x = N_y$ the sum of all of the resonators is $N = N_x\ N_y = N_x^2 = N_y^2$. Hence, for such a case, the panel surface area of the Helmholtz resonators S_p equals to:

$$S_p = (N_x - 1)d_{HR}(N_y - 1)d_{HR}$$
$$S_p = (\sqrt{N} - 1)^2 d_{HR}^2 \tag{18}$$

Thus, the following restriction needs to be imposed in order for the model to be accurate in physical terms:

$$2r_{HR} < d_{\mathrm{HR}} \Rightarrow 2\sqrt{\frac{S_{HR}}{\pi}} < d_{\mathrm{HR}} \Rightarrow S_{HR} < \frac{d_{HR}^2 \pi}{4}$$

As with the previous case, for such a model, an omnidirectional point source and a receiver point were placed at the Cartesian coordinates A (x_A, y_A, z_A) and C (x_C, y_C, z_C) respectively. The omnidirectional point sound source is assuming to emit a sound pressure and the first signal that reaches the receiver ("blue" solid line, Fig. 5) is the direct sound from the source. The time delay (in samples, for discrete time analysis) and the pressure attenuation, due to the distance in the air medium, e.g., from point A to point C, are calculated by the following equations:

$$n_{AC} = \frac{\sqrt{(\mathrm{x_C} - \mathrm{x_A})^2 + (\mathrm{y_C} - \mathrm{y_A})^2 + (\mathrm{z_C} - \mathrm{z_A})^2}}{c} F_s \tag{19}$$

$$p_C = \frac{p_A}{\sqrt{(\mathrm{x_C} - \mathrm{x_A})^2 + (\mathrm{y_C} - \mathrm{y_A})^2 + (\mathrm{z_C} - \mathrm{z_A})^2}} \tag{20}$$

where p_A is assumed to be a source reference pressure.

For the sound reflected from the panel, taking into account the finite distance differences from the source to each resonator in the panel, the specific time delay and the sound pressure attenuation are calculated for each resonator element, so that the input sound that reaches each resonator is attenuated by $\frac{p_A}{r_{ABi}}$ and delayed by n_{ABi} samples, where i is the index number of each Helmholtz resonator. In the next step the incoming sound pressure is filtered by the Helmholtz resonator filter $h_{HR}(n)$ response (Eq. (5)) and the output sound pressure from each resonator is represented by an omnidirectional wave, emitted to the receiver (dashed line, Fig. 5).

Hence, the combined response of all resonator elements in the array will be given as:

$$h_{THRC}(n) = \sum_{i=1}^{N} \frac{1}{r_{ABi} + r_{BCi}} h_{HRC}(n - (n_{ABi} + n_{BCi})) \tag{21}$$

where $h_{THRC}(n)$ is the combined response of all Helmholtz resonators at the receiver (point C). Note that in all simulations the interference from each resonator to another occur only as combined signals in the receiver point.

As for the previous analysis, it can be also assumed that, the pressure reflected from the rest of the panel's surface is emitted as a distinct (specular) reflection, i.e., from the panel filtered image of the source ("pink" doted line). To consider this panel's contribution at the receiver (point C), expressed by its impulse response $h_{PC}(n)$ the combined array-panel system will be characterized by the combination of the two linear systems, i.e., the response $h_{THRPC}(n)$ of the resonator array and panel system will be:

$$h_{THRPC}(n) = h_{THRC}(n) * h_{PC}(n) \tag{22}$$

where $*$ denotes the convolution symbol.

The complete impulse response of the system including the direct path component at the receiver will be:

$$h'_{THRPC}(n) = \frac{1}{r_{AC}} \delta(n - n_{AC}) + c_f\, [h_{THRC}(n) * h_{PC}(n)] \tag{23}$$

where c_f is a calibration factor that depends on *N* to account for the combined effect of *N* signals from the Helmholtz resonators arriving at the receiver. Due to the summation of *N* elements for the HR path, the overall level of the response $h_{THRC}(n)$ (see Fig. 6) increases disproportionally with respect to the level of the reflection that would be received due to a normal panel (e.g., the level of $h_{PC}(n)$). The calibration factor adjusts the peak level of the combined panel and Helmholtz Resonators, to the level of a non perforated panel. The complete block diagram of the processing steps followed is shown in Fig. 6.

Typical time and frequency domain response results for a 4 × 4 panel absorber attached to a panel is shown in Fig. 7. For this example, the HR filter parameters were $r_{HR} = 0.005$ m, $V_{HR} = 0.0003024$ m^3, $L_{HR} = 0.05$ m so $f_0 \simeq 115$ Hz, and $d_{\mathrm{HR}} = 0.08$ m, whereas for the panel surface, with typical absorption coefficient was considered in order to illustrate more clearly the panel's material effect, having 1/3 octave absorption coefficients shown in Table 1.

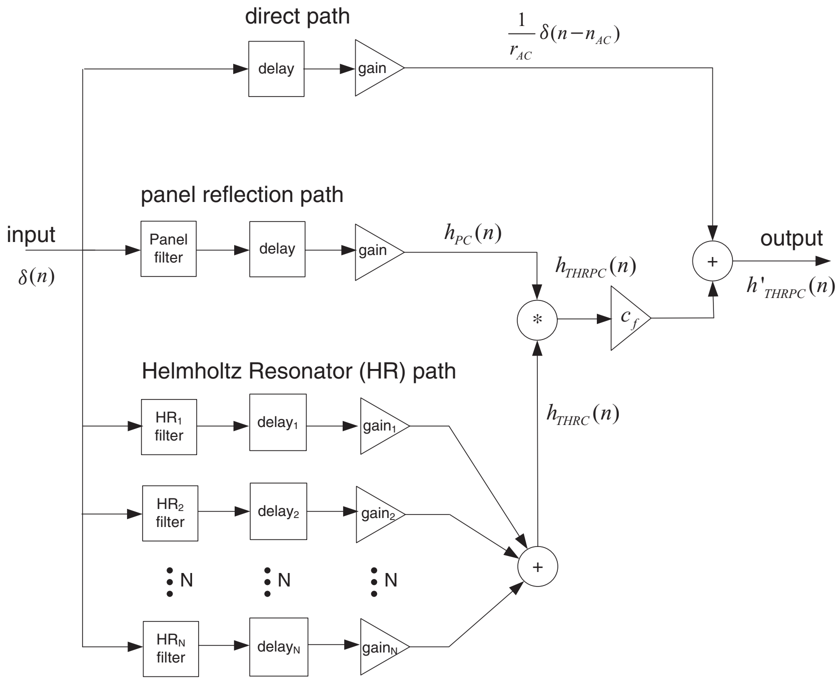


Fig. 6. Block diagram for the proposed filter-based model of an absorber panel containing an array of N Helmholtz resonators. Here, the ideal free field model is shown, whereas for the diffuse field case, the input will be the ideal diffuse field impulse responses and the direct path will be omitted (see Section 2.5).

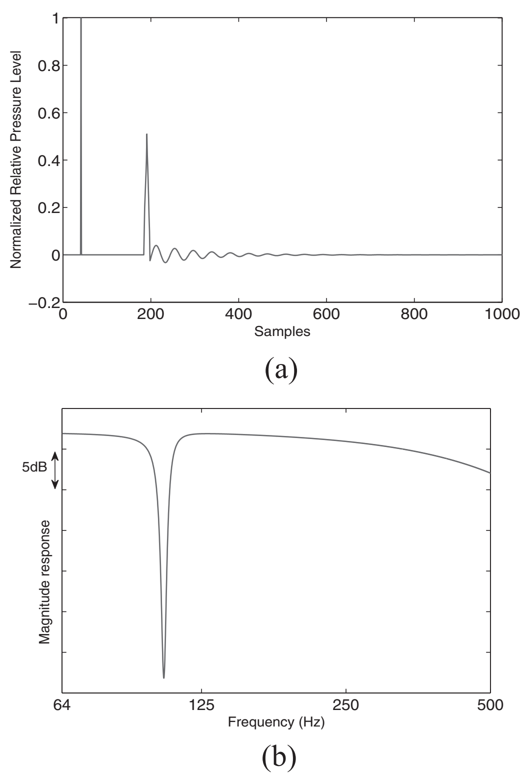


Fig. 7. Impulse response at receiver position C (0, 1, 2) for a combined 4×4 Helmholtz resonators array and panel at B (0, 0, 0) due to a delta impulse at source position A (0, 0, 1) and $f_0 = 120$ Hz. (a) Impulse response, (b) Magnitude frequency response.

Table 1. Absorption coefficient values in 1/3 octave bands (up to 500 Hz shown) for the simulated panel surface.

frequency (Hz)	63	80	100	125	160
Absorption coefficient	0	0.01	0.02	0.04	0.06
frequency (Hz)	200	250	315	400	500
Absorption coefficient	0.09	0.12	0.15	0.18	0.2

2.5. Filter-Based Resonator Panel Model in Diffuse Field

The previous section was concerned with the filter-based evaluation of the resonator absorber panel's response in the free field. However, as was discussed in the Introduction, the absorbing properties of such panels need to be measured via the reduction of sound energy achieved under ideal diffuse field conditions. Ideally such conditions dictate random instances of arrival times and angles of incidence for the reflected components. Following the proposed model such a condition will be realized assuming that the sound input of the simulated panel system is not an ideal Delta function (Eq. (23)), but instead it is an ideal diffuse field response. By definition, such an impulse response will be characterized by randomly arriving reflections, decaying exponentially with time. Such a diffuse component of room impulse responses $h_r(t)$ may be modeled by exponentially decaying white noise [23], i.e.,

$$h_r(t) = w(t)e^{-\xi_r t} \tag{24}$$

where $w(t)$is the white noise sequence, $\xi_r = \frac{3\ln 10}{T_r}$ and T_r (sec) is the reverberation time of the enclosure. In discrete time, such ideal diffuse field room response is expressed as:

$$h_r(n) = w(n)e^{-\xi_r n} \tag{25}$$

Given that in such an ideal diffuse field the direct path contribution needs to be minimized, the complete response of the absorber panel system (Eq. (23)) will be written as:

$$h''_{THRPC}(n) = [c_f h_{THRPC}(n)] * h_r(n) \tag{26}$$

Hence, by driving the panel filter model via such diffuse field response corresponding to an ideal reverberation chamber (Eq. (26)), the filtered output will emulate the measured response at a specified receiver position, when the absorber panel is installed. In this case the modeling procedure will also follow the block diagram shown in Fig. 6, the input being the ideal diffuse room response $h_r(n)$ (see Eq. (25)) and output will be the response $h''_{THRPC}(n)$ (see Eq. (26)). Furthermore, the direct path component will be omitted.

2.6. The Absorption Coefficient of Resonator Panel Absorbers

In order to evaluate the absorption properties of panels modeled according to the proposed approach, an ideal diffuse reverberant chamber is simulated whose impulse response (arriving at the resonator-panel system) is evaluated according to Eq. (25) and for arbitrarily chosen value of the Reverberation Time T_r. Then, any resonator absorption panel of specified parameters (perforation spacing, resonant frequency, panel material, and size) is assumed to be placed inside such space, resulting to a modified response $h''_{THRPC}(n)$ (see Eq. (26)) at a receiver position and now having a modified reverberation time $T_p$due to increased absorption. The simulated tests here attempt to predict the absorption coefficient of the panel system under such ideal diffuse field conditions, as is dictated by the standardized procedure.

For the subsequent analysis and tests, following the ISO 354 standard recommendation [13], 1/3 octave bandpass filters were applied to the response $h''_{THRPC}(n)$ in order to evaluate the $T_p$in the corresponding frequency bands. As is also required by the ISO 354 standard, a repeated sequence of ideal diffuse responses are driven into the simulated system and the fractional 1/3 octave output signals are averaged to produce the estimate of the bandlimited pressure response.

Typical results are shown in Fig. 8. For these simulations, the response of a resonator panel with $N = 2500$, $S_p = 2$ m^2 and resonance frequency $f_0 = 120$ Hz was evaluated placed in a simulated reverberant chamber with Reverberation Time $T_r = 3$ sec and volume $V_r = 150$ m^3. The averaged 1/3 octave frequency filter output due to the modified room response $h''_{THRPC}(n)$, focusing on the resonator active region, is shown in Fig. 8(a). Note that for these tests, the panel surface material (without the resonators) was assumed to have frequency-independent absorption coefficient of $\overline{a_P} = 0.1$.

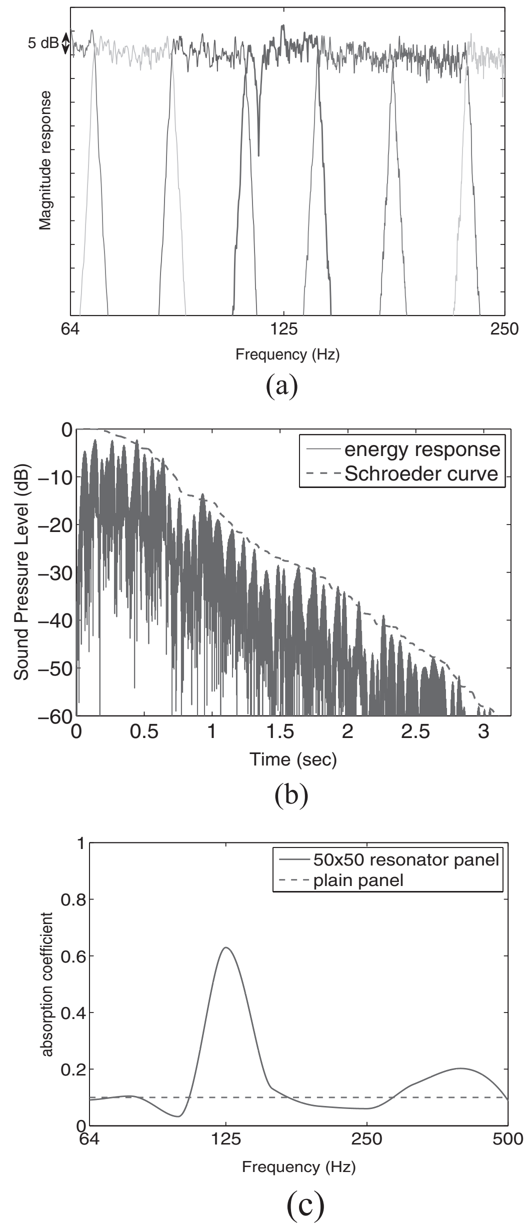


Fig. 8. Typical results for diffuse field simulation for a 50×50 resonator panel. The resonance frequency of each resonator was $f_0 = 120$ Hz and the room reverberation time was $T_r = 3$ sec. A frequency independent absorption coefficient $\overline{a_P} = 0.1$ was assumed for the plain surface of the panel. (a) simulated averaged pressure output of the 1/3 octave filter bank (note that only the relevant bands are shown around the resonance frequency, f_0,) (b) time energy 1/3 filter output response at the band of the resonance (in dB) derived from the ideal diffuse field response $h''_{THRPC}(n)$, and corresponding Schroeder 1/3 octave reverberation decay curve. (c) estimated frequency-dependent absorption coefficient of the combined resonator array and panel (see Section 2.6).

From each bandlimited response output, the Reverberation time T_p was evaluated according to the Schroeder's integration method [24], as is shown in Fig. 8(b).

For the estimation of the panel's absorption coefficient let us assume that we start with an ideal diffuse chamber of volume V_r (m^3), total surface area S_r (m^2), and, for

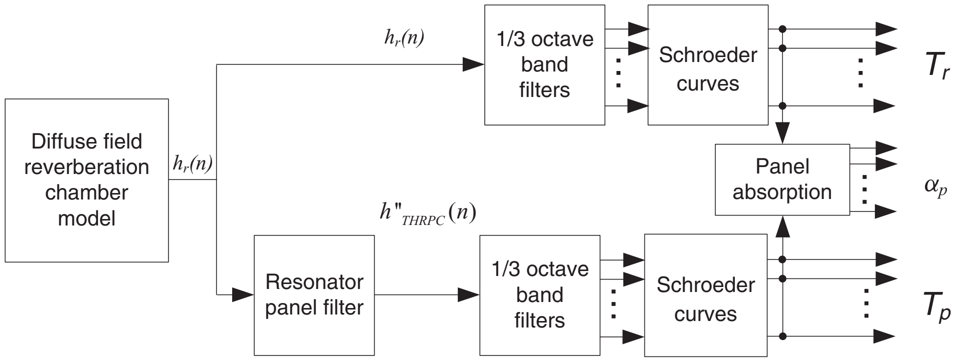


Fig. 9. Block diagram illustrating the steps for simulating diffuse field reverberation chamber response (Eq. (24)), the effect of the panel in the chamber (Eq. (26)), the 1/3 octave analysis of the corresponding responses and the estimation of the reverberation time, and the panel's absorption coefficient (Eq. (28)).

simplicity, the Reverberation Time according to Sabine's formula is given by [25]:

$$T_r = \frac{0.161 V_r}{A_s}, \text{ where } A_s = \sum_i S_{r_i} a \tag{27}$$

where a is the uniform Sabine absorption coefficient and A_s (m^2) is the total absorption of the chamber.

If the perforated resonator panel modeled as before with surface area S_p (m^2) is placed on a wall within this chamber, then the Reverberation Time will be changed to T_p since the mean absorption coefficient will be affected by the additional absorption. According to ISO 354 recommendation, the absorption coefficient of this additional surface can be calculated from the equivalent sound absorption area, A:

$$A = 55.3\frac{V_r}{c}\left(\frac{1}{T_p} - \frac{1}{T_r}\right), a_p = \frac{A}{S_p} \tag{28}$$

where a_P is the panel's absorption coefficient. Note that according to the ISO standard, this analysis refers to 1/3 octave filtered signals. As was also described in the previous section, this was the procedure followed for the typical example of Fig. 8, assuming here a typical plain panel surface having uniform with frequency area absorption $\overline{a_P} = 0.1$ and the results being estimated for combining this panel to the 50×50 resonator array. The evaluated absorption coefficient for the combined resonator-panel system is shown in Fig. 8(c).

The complete procedure for the proposed filter-based estimation of the absorption coefficient of the resonant panel absorber is shown in Fig. 9.

3 RESULTS

3.1 Comparison to Measured Data

As was discussed in the previous sections, the proposed approach allows efficient evaluation of resonator panel absorber response either for free field or diffuse field conditions. Although potential applications of such simulation results may be applicable to other areas of room acoustics, here indicative results obtained by the proposed method following the ISO 354 recommendation procedure discussed in Sections 2.5 and 2.6 will be compared to published data for a commercial panel absorption coefficient measured in-situ [26].

Table 2. Absorption coefficient values in octave for the simulated plywood panel surface.

frequency band (Hz)	125	250	500	1000	2000	4000
Absorption coefficient	0.28	0.2	0.1	0.1	0.08	0.08

For this comparison, the simulated case was using panel properties derived from data provided from the particular product, as were published by the manufacturer [26]. From these published data, a total panel specimen area $S_P = 12m^2$ having a perforation ratio of 1.56% was specified, indicating a total area for the resonator elements of $S_{HR} = 25$ mm^2. Given that the panel thickness was $L'_{HR} = 16$ mm and placed at a distance of 50 mm from the wall, the total volume of the $N = 7596$ resonator elements was estimated, leading to a resonant frequency of $f_0 = 242$ Hz for each resonator. For the plain panel surface, the panel filter response was adjusted in order to follow the absorption coefficient values corresponding to the specimen material, i.e., assumed to be plywood, as given in Table 2.

These test parameters were employed for the diffuse field response evaluation of the panel specimen response (see Sections 2.5 and 2.6) assuming also a reverberant chamber of $V_r = 185m^3$ and $S_r = 194m^2$ (as for published measurement set-up). For averaging the responses, 800 runs of the simulated filtering procedure were computed as described in the previous Sections. The comparative 1/3 octave results for the measured and estimated absorption coefficient are shown in Fig. 10 indicating a close approximation between measured and simulated data, especially around the resonant frequency region. Some discrepancy is observed at higher frequencies and can be possibly due to inexact representation of the specific plain surface absorption of the specific panel (Table 2). Such discrepancies can be reduced when the exact properties of the panel material are known prior to modeling.

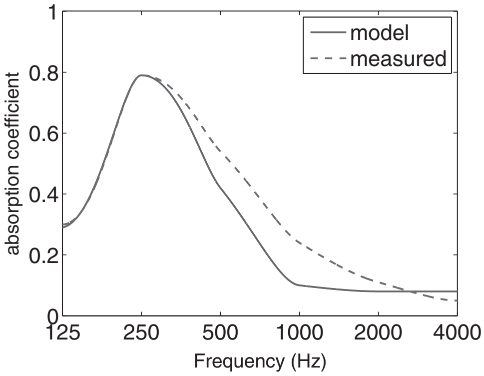


Fig. 10. Typical comparative results for absorption coefficient evaluated for 1/3 octave bands of a commercially available resonator absorption panel and the simulated data of the same panel obtained by the proposed method.

Table 3. Typical CPU computation time as function of N resonator elements for the numerical evaluation of resonator panel response using a FEM-based model and the proposed filter-based model (for 800 runs). In both cases, the evaluations were performed up to a maximum frequency of 500 Hz.

	$N = 1$	$N = 16$	$N = 64$	$N = 2500$
FEM-based model	4 hours	17 hours	2 days	Not feasible
Filter-based model	50 sec	4 mins	15 mins	20 hours
	$N = 1$	$N = 16$	$N = 64$	$N = 2500$

3.2 Computational Complexity

As was discussed in the Introduction, the proposed filter-based approach provides an efficient alternative for simulation of resonator absorption panel systems. Typical comparison of the CPU time required for the numerical estimation using a FEM based model [11] and the proposed filter-based model running on Matlab [27] and for different number of resonator elements (*N*), is shown in Table 3. Note that when following the proposed method, an impulse response of the simulated system is directly evaluated, leading to further simplification of subsequent spectral system properties.

4 CONCLUSIONS

A novel and computational efficient method for evaluating the response of a perforated absorption panel with an arbitrary number of Helmholtz resonator elements has been proposed based on a simplified, parametric filter-based model. The model is based on 2^{nd} order parametric IIR digital filters that emulate the linear filtering response of each resonator and on the evaluation of the relative acoustic path delays and gains of such responses, as received at any point due to an arbitrary placed ideal excitation. The combined response of these resonators and of a panel surface having user-defined reflection properties can be also predicted, leading to the efficient evaluation of the system's impulse and frequency response functions. In effect, such an approach leads to a simplified, but reasonably accurate and fast evaluation of the response of perforated absorption panels for any specified type of resonator, perforation and panel surface properties, at any receiver position under ideal free field conditions. An extension of the method, allows the evaluation of the response of such a system under ideally diffuse acoustic excitation and an efficient estimation of the resulting time, energy, and frequency response. From those functions, the reduction of the initial diffuse field reverberation time due to the panel absorption can be evaluated, leading to the estimation of the frequency-dependent absorption coefficient of the simulated panel specimen according to the ISO standard. This was confirmed by a comparative test between simulated results obtained by the proposed method and published measurement data derived from a standardized test of a commercially available perforated panel.

The proposed methodology follows some necessary simplifications, for example disregarding any angle of incidence-dependent absorption effects of panels and resonators, any coupling between the resonators, scattering effects, the analytic properties of sound radiation from such cavities, etc. Nevertheless, given that analytic solutions for systems and applications of complexity such as of the ones covered by the proposed approach are beyond the capabilities of current computer systems, the proposed method introduces a flexible and practical alternative having far shorter and manageable computation time requirements than any FEM-based method.

A particular advantage of the proposed approach is that it can be readily integrated with current room acoustic simulation methods based on geometric acoustics. For such methods, any desired perforated panel and HR absorber may be attached to the virtual room boundaries, leading to evaluation of modified impulse responses due to the addition of absorption. Similarly, the resulting effect of adding panel absorbers on the acoustic indices may be also easily and more accurately evaluated than by using the usual look-up tables of panel material absorption coefficients. Furthermore, the acoustic effect of the addition of perforated panels may be also evaluated via listening tests (auralization), since binaurally processed impulse responses of these panels may be easily emulated at the desired listening position.

Although not evaluated during this work, it is also feasible that the proposed method may be useful for estimating the distance and angle depended response of any arbitrary-sized perforated panel surface, not only having a given perforation ratio but also potentially having any non-symmetric distribution of the resonator element array. Furthermore, the resonator elements may be unequally-sized, potentially leading to novel and optimized solutions for sound absorption. Such aspects will be considered in future studies.

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